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EFFECT OF ELECTRIC CURRENT ON THE EVOLUTION OF PLASTIC STRAIN NEAR A CRACK TIP

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The effect of direct current on the evolution of plastic strain near the tip of a crack in a crystal in tension is studied. The plastic strain near the crack tip in the loaded specimen is the result of the motion of dislocations in the active slip planes of the crystal under the action of shear stresses caused by external loading and electric current. The Joule heat, Thomson effect, and "electron wind" (electroplastic effect) are taken into account in calculations. The plastic strain and stress distributions near the crack tip are determined at different moments of time for a given magnitude of electric current. The effect of the plastic zone on the stress-intensity factor of the crack is studied. It is found that the plastic strain is affected largely by the Joule heat released upon passage of the electric current. A numerical analysis is performed for an α-Fe crystal.

Introduction. It is known that high-density current in a metal substantially decreases its deformation resistance and increases its plasticity. A large body of research [1, 2] shows that the following factors affect the plastic-strain evolution in a solid body upon passage of electric current: 1) Joule heating; 2) electroplastic effect (EPE); 3) pinch effect (action of an inherent magnetic field). Experiments [2] show that the EPE plays the main role in the plastic-strain evolution, and in this process, the plastic strain has a thermofluctuational character and the action of current is equivalent to the action of mechanical stresses. We note that the latter statement is disproved in [3].

The effect of electric current on the evolution of plastic strain near the tip of a crack is of interest, since the stress-intensity factor (SIF) depends on plastic strain. Maksimov and Svirina [4, 5] studied the effect of the Joule heat on the growth of a crack with a plastic zone near its tip in a loaded model specimen. However, other effects of electric current and the microstructure of plastic strain were ignored in these studies.

The aim of the present work is to calculate the plastic strain near the tip of a crack in a crystal and its SIF under the action of a tensile force and direct electric current. The effect of the Joule heat, EPE, and Thomson effect (displacement of the heating zone in the direction of the motion of electrons) are taken into account.

1. Formulation of the Problem and Method of the Solution. The authors [6, 7] formulated the problem of plastic deformation of a body-centered cubic lattice of a crystal near the tip of a crack of length $2l$ located in the cleavage plane (010) at an ambient temperature T_0. It was assumed that the cracked crystal is loaded by a tensile stress $\sigma'_a(t)$ applied far from the crack. The plastic strain $\varepsilon_j(\boldsymbol{r}, t)$ occurred due to displacement of the total dislocations with the Burgers vector $(1/2)\langle 111\rangle$ over two $\{110\}$ planes of easy slip [$(\bar{1}10)$ for $j = 1$ and (110) for $j = 2$] in the $\boldsymbol{\xi}_j$ directions for the corresponding planes (Fig. 1).

In addition to [6, 7], we assume that, at a distance from the crack, electric current of density $J'_0(t)$ passes through the crystal in the normal direction to the crack plane. We also assume that the quantity $J'_0(t)$ increases monotonically up to a certain value J_0 in the time t_0 and then it remains constant, and the effect of electric current on the cracked crystal is confined by the first and second mechanisms. In this case, the equation for the effective stress $\sigma^e_j(\boldsymbol{r}, t)$ (2.3) in [6] should involve the shear stress $\sigma^t_j(\boldsymbol{r}, t)$ due to EPE, which is determined by the formula [2]

$$\sigma^t_j(\boldsymbol{r}, t) = B_e \boldsymbol{J}(\boldsymbol{r}, t) \cdot \boldsymbol{\xi}_j/(ben), \tag{1}$$

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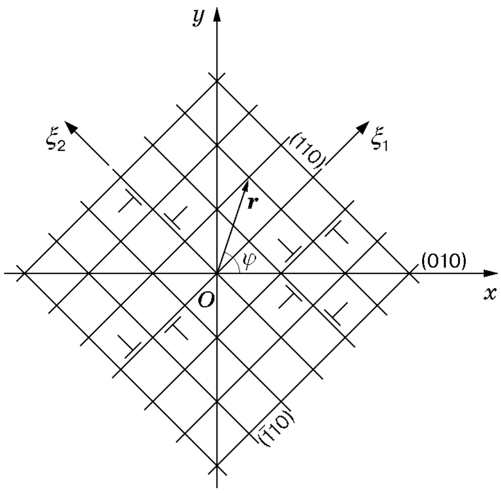

Fig. 1. Crystallographic scheme of a crack and planes of easy slip in a body-centered cubic lattice in a crystal in tension.

where B_e is a constant, n is the concentration of free electrons in the crystal, and e is the elementary charge. From [8] follows the expression for the vector of the electric-current density $\boldsymbol{J}(\boldsymbol{r},t)$ near the crack tip in polar coordinates $J_x(\boldsymbol{r},t) = J_0'(t)\sqrt{l/(2r)}\,\sin(\varphi/2)$ and $J_y(\boldsymbol{r},t) = J_0'(t)\sqrt{l/(2r)}\,\cos(\varphi/2)$.

We consider the heat released in the plastic zone near the crack tip. For the strain rate $\dot\varepsilon_j(\boldsymbol{r},t)$ in (2.1) (see [6]), the temperature field in the upper half-plane at the time t is determined by the formula [9]

$$T(x,y,t) = \int\limits_0^\infty dy \int\limits_{-\infty}^\infty T_0(x,y) G_2(x,y,x_1,y_1,t)\,dx_1$$

$$- a\int\limits_0^\infty d\tau \int\limits_{-\infty}^\infty \Phi(x_1,\tau) G_2(x,y,x_1,0,t-\tau)\,dx_1$$

$$+ \int\limits_0^\infty d\tau \int\limits_0^\infty dy_1 \int\limits_{-\infty}^\infty F(x_1,y_1,\tau) G_2(x,y,x_1,y_1,t-\tau)\,dx_1 \qquad (2)$$

as the classical solution of the heat-conduction problem

$$\frac{\partial T}{\partial t} = a\Delta T + F(x,y,t), \qquad -\infty < x < \infty, \quad 0 \leqslant y < \infty; \qquad (3)$$

$$\frac{\partial T(x,0,t)}{\partial y} = \Phi(x,t); \qquad (4)$$

$$T(x,y,0) = T_0(x,y), \qquad 0 < t < \infty. \qquad (5)$$

In (2), $G_2(x,y;x_1,y_1,t) = G(x,y;x_1,y_1,t) + G(x,y;x_1,-y_1,t)$, $G(x,y;x_1,y_1,t) = 1/(4\pi a t)\exp\{-[(x-x_1)^2 + (y-y_1)^2]/(4at)\}$, $a = \lambda/(\rho C)$ is the thermal diffusivity of the crystal, λ is the thermal conductivity, ρ is the density, and C is the specific heat. In (2) and (3),

$$F(x,y,t) = \sum_{j=1}^{2}\Big[\sigma_j^e(x,y,t)\dot\varepsilon_j(x,y,t)\Big] + \varrho J^2(x,y,t) + L J_i(x,y,t)\nabla_i T(x,y,t). \qquad (6)$$

Here $\varrho = \varrho_0[1 + \alpha(T - T_0')]$ is the specific electrical resistance of the crystal, α is the temperature coefficient of electrical resistance, $T_0' = 273$ K, and L is the Thomson's coefficient [8]. The symmetry of the problem about the Ox axis implies the boundary condition $\Phi(x,t) = 0$. In the numerical solution of (2), the quantities $T(x_1,y_1,t)$ and $F(x_1,y_1,t)$ are assumed to be constant at each time step in square cells in the Oxy plane, whose centers are the nodes of the coordinate grid and whose side is equal to the size of the grid h. In this case, the desired temperature $T(x, y, t)$ is expressed in terms of the integrals

$$I_1 = I_1^+ + I_2^-, \quad I_2 = I_2^+ + I_2^-, \quad I_1^\pm = \int\limits_\Omega G(x, y, x_1', \pm y_1', t)\, dx_1'\, dy_1', \quad I_2^\pm = \int\limits_\Omega I_3^\pm\, dx_1'\, dy_1',$$

$$I_3^\pm = \int\limits_0^t G(x, y, x_1', \pm y_1', t-\tau)\, d\tau = \frac{1}{4\pi\lambda}\Big[\mp \mathrm{Ei}\Big(-\frac{(x-x_1')^2 + (y \mp y_1')^2}{4a^2 t}\Big)\Big],$$

where $\Omega = h^2$ is the area of the cell of the calculation grid and $\mathrm{Ei}(-X)$ is the integral exponent function. We replace $\mathrm{Ei}(-X)$ by its asymptotic representation $\mathrm{Ei}(-X) = -\ln X - \gamma$ for $X \ll 1$, where $\gamma = 0.5772\ldots$ is the Euler constant. Then, we obtain

$$I_2^\pm = \Psi(u_2, v_2) - \Psi(u_2, v_1) - \Psi(u_1, v_2) + \Psi(u_1, v_1),$$

$$\Psi_1 = 4a^2 t\Big[(3-\gamma)uv - uv\ln(u^2+v^2) - \frac{3u^2+v^2}{2}\arctan\Big(\frac{v}{u}\Big) - \frac{3v^2+u^2}{2}\arctan\Big(\frac{u}{v}\Big)\Big],$$

where $u_1 = (x - (x_1 - h/2))/\beta$, $u_2 = (x - (x_1 + h/2))/\beta$, $v_1 = (y - (\pm y_1 - h/2))/\beta$, $v_2 = (y - (\pm y_1 + h/2))/\beta$, $\beta^2 = 4at$, and $I_1 \simeq h^2/(2\pi at)$.

Equations (2.1)–(2.8) from [6] with formulas (1) for $\sigma_j^t(\boldsymbol{r}, t)$ and (2) for $T(\boldsymbol{r}, t)$, form a system from which $\varepsilon_j(\boldsymbol{r}, t)$, $\sigma_j^e(\boldsymbol{r}, t)$, and $T(\boldsymbol{r}, t)$ are determined for the initial conditions

$$\varepsilon_j(\boldsymbol{r}, 0) = 0, \qquad \sigma_a'(0) = 0, \qquad T(\boldsymbol{r}, 0) = T_0 \tag{7}$$

and the boundary conditions

$$\sigma_j^e(x, 0, t) = 0 \quad (x < 0), \qquad \frac{\partial T}{\partial y}(x, 0, t) = 0. \tag{8}$$

This system was solved numerically with a variable integration step Δt by the method used in [6]. The time step was chosen with allowance for the restrictions imposed on the maximum strain rate $\max|\dot{\varepsilon}_j(r, t)| \leqslant 0.1\ \mathrm{sec}^{-1}$. Under this restriction, formula (2.1) in [6], which corresponds to thermal activation of the motion of dislocations, remains valid.

We assume that the SIF of the crack $K(t)$ can be written in the form [6]

$$K(t) = K^c(t) + K^p(t), \tag{9}$$

where $K(t)$ is the SIF of the crack "dressed in a dislocation coat," i.e., surrounded by dislocations (the effect of plastic strain on the crack is taken into account), $K^c(t)$ is the SIF of the "naked" crack (the plastic strain near its tip is ignored) for extension by an external load (mode I), and $K^p(t)$ is the correction to $K^c(t)$ due to the effect of the "dislocation coat" (plastic strain) near the crack tip. The quantity $K^p(t)$ is determined by the formula [6]

$$K^p(t) = \sum_{j=1}^{2}\int\limits_{D_j} \hat{K}^p(z, j)\Delta\rho_j(z, t)\, dz, \qquad z = x + iy, \tag{10}$$

where D_j $(j = 1, 2)$ are the plastic-strain zones near the crack tip that occur due to the motion of dislocations over the planes of easy slip, $\Delta\rho_j(z, t)$ is the density of effective dislocations in these planes, and $\hat{K}^p$ is the complex SIF of the crack loaded by the stress induced by an effective dislocation located in the upper and lower half-planes. In this case, we express $\hat{K}^p = \hat{K}^p_{\mathrm{I}} - i\hat{K}^p_{\mathrm{II}}$ by the formula [6]

$$K^p_{\mathrm{I}}(z, j) - iK^p_{\mathrm{II}}(z, j) = \frac{A}{\sqrt{\pi}}\,[J_1 + iJ_2(-1)^j],$$

where

$$A = \frac{Gb}{2\pi(1-\nu)},\ J_1 = -\pi\Big[\frac{1}{\sqrt{z}} + \frac{3}{2\sqrt{z}} - \frac{z}{2(\bar{z})^{3/2}}\Big],\ J_2 = -\pi\Big[\frac{1}{\sqrt{z}} + \frac{1}{2\sqrt{\bar{z}}} + \frac{z}{2(\bar{z})^{3/2}}\Big],\ \bar{z} = x - iy.$$

Here $\hat{K}^p_{\mathrm{I}}$ is the correction to $K^c(t)$ for extension (mode I) and $\hat{K}^p_{\mathrm{II}}$ is the correction to $K^c(t)$ for shear (mode II).

2. Calculation Results and Discussion. Calculations were performed for an α-Fe crystal with the following constants: $\rho = 7800$ kg/m^3, $\lambda = 78.2$ W/(m $\cdot$ K), $C = 460$ J/(kg $\cdot$ K), $T_0 = 300$ K, $\varrho_0 = 8.6 \cdot 10^{-8}$ $\Omega \cdot$ m,

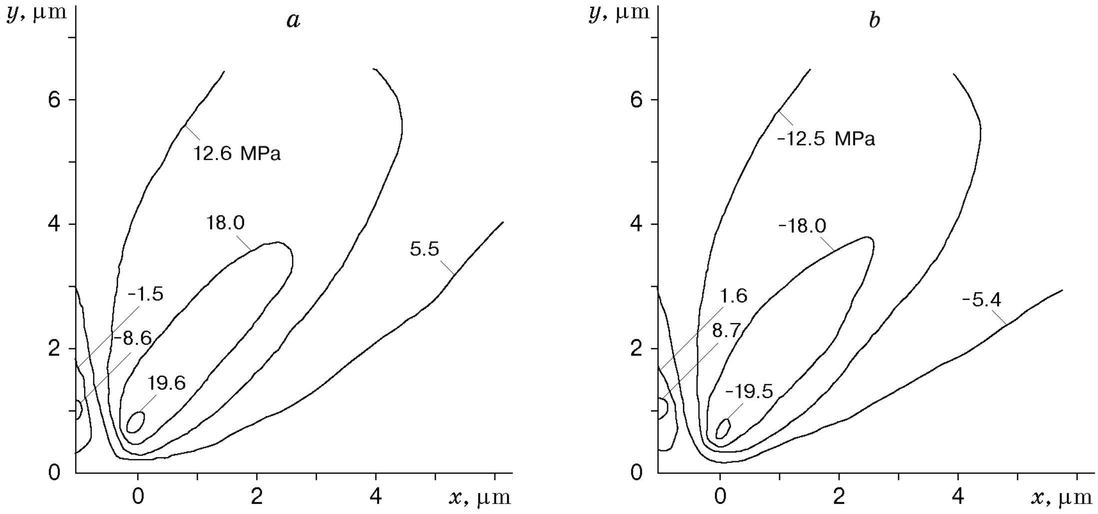

Fig. 2. Distribution of the effective shear stress near the crack tip over the slip planes $(\bar{1}10)$ (a) and (110) (b) for direct current of density $J_0 = 10^9$ A/m^2 at $t = 1.05$ sec.

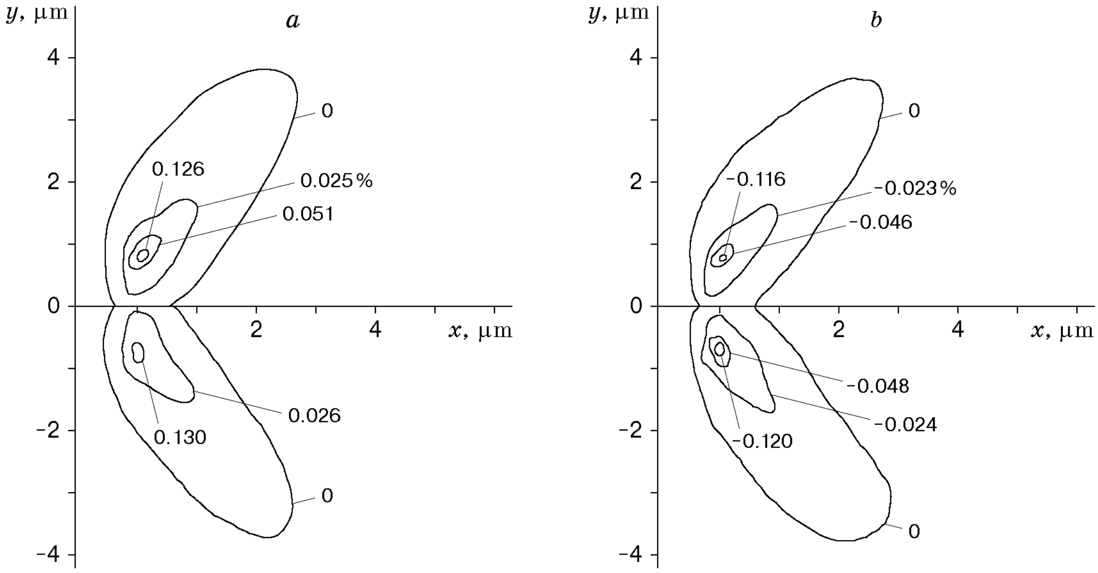

Fig. 3. Distribution of the shear strain near the crack tip over the slip planes $(\bar{1}10)$ (a) and (110) (b) for direct current of density $J_0 = 10^9$ A/m^2 at $t = 2.25$ sec.

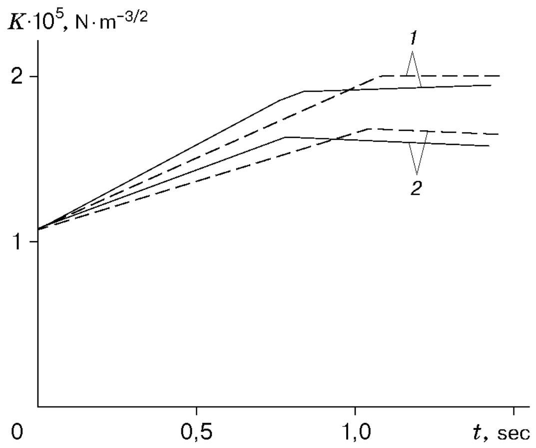

Fig. 4. Time dependences of the SIF of a “naked” crack $K^c(t)$ and $K(t)$, calculated with allowance for plastic strain near its tip: the solid curves refer to $J_0 = 0$ and the dashed curves to $J_0 = 10^9$ A/m^2; curves 1 refer to K^c and curves 2 to K.

$L = -22.8 \cdot 10^{-6}$ V/K, $B_e = 10^{-5}$ Pa/sec, $\alpha = 3.3 \cdot 10^{-3}$ K^{-1} [10], $t_0 = 0.25$ sec, $J_0 = 10^9$ A/m^2, and $2l = 10^{-3}$ m. The remaining constants were borrowed from [6]. The external load $\sigma'_a(t)$ was assumed to reach the upper limit $\sigma_a = 5$ MPa and then remain constant. In this stage of calculations, we studied the relaxation of the effective shear stresses $\sigma^e_j(\boldsymbol{r}, t)$, which decelerates the evolution of $\varepsilon_j(\boldsymbol{r}, t)$ up to its cessation.

The plastic-strain evolution in the absence of electric current is studied in detail in [6, 7]. We compare the plastic-strain evolution in the absence of electric current with that with allowance for the Joule heat, Thomson effect, and “electron wind” (EPE). The loading stage takes 0.79 and 1.05 sec in the first and second cases, respectively. For a constant external load and in the absence of electric current, the stress relaxation is completed at the time $t = 1.32$ sec ($\varepsilon_{\max} = 0.125\%$), whereas the electric current terminates the plastic-strain evolution at the moment $t = 2.25$ sec.

Figure 2 shows the distribution of the effective shear stress $\sigma^e_j(\boldsymbol{r}, t)$ near the crack tip over the slip planes $(\bar{1}10)$ (Fig 2a) and (110) (Fig. 2b) for direct electric current at $t = 1.05$ sec. Figure 3 shows the distribution of the plastic strain over the slip planes $(\bar{1}10)$ (Fig. 3a) and (110) (Fig. 3b) after stress relaxation is completed at $t = 2.25$ sec. An analysis of the effect of the “electron wind” shows that it decelerates stress relaxation in the plastic zone in comparison with a current-free regime. It follows from the results of calculation of stresses and strains that the time it takes for the external load to reach a maximum with allowance for the Thomson effect exceeds that without allowance for this effect.

Figure 4 shows the time dependences of the SIF calculated with and without allowance for electric current. In all the cases considered, a decrease in the SIF after stress relaxation due to plastic strain near the crack tip is 17.6%. We also note that, in this case of calculation of the Joule heat, the contribution of the first term in (6) is negligible compared to the contribution of the other two terms.

The present analysis has shown that:

1) during loading of the crystal, the action of direct electric current decreases the stress rate near the crack tip for a given maximum plastic-strain rate;

2) the plastic-strain evolution is affected mainly by the Joule heat released, the influence of the Thomson effect is also pronounced, and the “electron wind” leads to asymmetry in the plastic-strain distribution of the in different slip planes.

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